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TRANSIENT RADIATIVE-CONDUCTIVE HEAT TRANSFER IN A FLAT LAYER OF A SELECTIVELY ABSORBING AND RADIATING MEDIUM

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Results of a numerical solution of the unsteady boundary-value problem of radiative-conductive heat transfer in a flat layer of a selective nonscattering medium with semitransparent mirror-reflecting boundaries are presented. This problem reduces to a nonlinear integral equation in the unknown temperature with the use of a Green function. The optical properties of the walls are shown to have a strong effect on the formation of a temperature field in the layer. The intensity of heating of the layer depends on the radiative fluxes to a greater extent than on the conductive fluxes.

The problem of joint heat transfer by heat conduction and radiation in various materials is associated with important engineering applications (heat transfer in fibrous insulators, heating and cooling of glasses, etc.). In view of this, a study of the contribution of radiation in the total heat transfer as applied to various physical and engineering problems is of keen practical interest.

The author analyzed [1] the unsteady selective problem of radiative-conductive heat transfer (RCHT) in a layer with semi-transparent boundaries, taking into account the temperature dependence of the absorption coefficient. The selective character of the radiation is taken into consideration either by the method of stepwise approximation of real absorption spectra [2, 3] or by averaging the absorption coefficient over the frequency [4, 5]. The need to perform a reasonable averaging of the volume absorption coefficient arises because of difficulties of a computational character that are connected with determination of the integral hemispherical density of the radiative flux.

Below, we formulate the RCHT problem of a semitransparent selectively absorbing and radiating medium that is separated by two mirror-reflecting parallel planes and we consider the method of solving this problem. The numerical algorithm for solving the problem assumes making allowance for the temperature dependence of the thermal and radiation characteristics of the medium. The mathematical formulation of the problem describes processes of heat transfer caused by heat conduction and radiation in the form of unsteady energy and transfer equations.

The energy equation with boundary conditions is written as follows:

$$\rho(T)c(T)\,\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(T)\,\frac{\partial T}{\partial x}\right] - \frac{\partial E}{\partial x}, \quad 0 < x < L, \quad t > 0; \tag{1}$$

$$\lambda\,\frac{\partial T}{\partial x} = \alpha_1(T - T_1) - \int\limits_{\Omega_1} \varepsilon_{1\nu}[Q_1(\nu, T_1^*) - E_{\nu 1}(\nu, T)]\,d\nu, \quad x = 0; \tag{2}$$

$$\lambda\,\frac{\partial T}{\partial x} = \alpha_2(T_2 - T) + \int\limits_{\Omega_2} \varepsilon_{2\nu}[Q_2(\nu, T_2^*) - E_{\nu 2}(\nu, T)]\,d\nu, \quad x = L; \tag{3}$$

$$T(x, 0) = T_0(x). \tag{4}$$

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Here

$$\frac{\partial E}{\partial x} = \int_0^\infty k_\nu [4\pi I_{\rho\nu} - G_\nu(x)]\, d\nu; \quad G_\nu(x) = 2\pi \int_0^1 [I_\nu^+(x,\mu) + I_\nu^-(x,-\mu)]\, d\mu;$$

T_1 and T_2 are the temperatures of the ambient medium, Q_1 and Q_2 are the external radiative fluxes, T_1^* and T_2^* are the temperatures of the external emitters, $E_{\nu i}$, $\varepsilon_{i\nu}$, and Ω_i are the densities of the fluxes of intrinsic radiation, degrees of emissivity, and spectral zones of nontransparency of the boundary surfaces, respectively.

The heat-transfer equations with boundary conditions are as follows:

$$\mu \frac{dI_\nu^+}{dx} + k_\nu I_\nu^+ = k_\nu I_{p\nu}(x); \tag{5}$$

$$\mu \frac{dI_\nu^-}{dx} - k_\nu I_\nu^- = -k_\nu I_{p\nu}(x); \tag{6}$$

$$I_\nu^+(0,\mu) = n_\nu^2[1 - R_{0\nu}(\mu)] I_{p\nu}(T) + R_{0\nu}(\mu) I_\nu^-(0,\mu); \tag{7}$$

$$I_\nu^-(1,\mu) = n_\nu^2[1 - R_{1\nu}(\mu)] I_{p\nu}(T) + R_{1\nu}(\mu) I_\nu^+(1,\mu). \tag{8}$$

Here x and k_ν are the dimensionless coordinate and the volume absorption coefficient, n_ν is the refractive index, and $R_{i\nu}$ are the reflection coefficients for the surface boundaries ($i = 1$ and 2).

After the new variable $u(x,t) = \int_0^\theta \lambda(z)\, dz$ is introduced, the boundary-value problem (1)–(4) takes the form

$$\frac{\partial^2 u}{\partial x^2} - u = F(q,x,t), \quad 0 < x < 1, \quad t > 0; \tag{9}$$

$$\frac{\partial u}{\partial x} = q_1, \quad x = 0; \tag{10}$$

$$\frac{\partial u}{\partial x} = q_2, \quad x = 1. \tag{11}$$

The formal solution of the boundary-value problem (9)–(11) with the use of the Green function for the differential operator on the left-hand side of Eq. (9) is as follows:

$$\int_0^\theta \lambda(z)\, dz = [q_2(\theta)\cosh(x) - q_1(\theta)\cosh(1-x)]/\sinh(1) + \int_0^1 F(\theta,z,t)\Gamma(x,z)\, dz, \tag{12}$$

where

$$F(\theta,x,t) = \sigma_0 T_*^3 G_R(x) + R(\theta)\frac{\partial\theta}{\partial t} - \int_0^\theta \lambda(z)\, dz; \quad G_R(x) = \frac{1}{\sigma_0 T_*^3}\int_0^\infty k_\nu[4\pi I_{p\nu}(\theta) - G(x)]\, d\nu;$$

$$q_1(\theta) = \alpha_1 L(\theta - \theta_1) - \sigma_0 T_*^3 L \int_{\Omega_1} \varepsilon_{\nu 1}[\Phi_1(\nu,\theta_1^*) - \Phi_{1\nu}(\nu,\theta)]\, d\nu \quad (x = 0);$$

$$q_2(\theta) = \alpha_2 L(\theta_2 - \theta) - \sigma_0 T_*^3 L \int_{\Omega_2} \varepsilon_{\nu 2}[\Phi_2(\nu,\theta_2^*) - \Phi_{2\nu}(\nu,\theta)]\, d\nu \quad (x = 1);$$

$$\Phi_i(\nu,\theta_i^*) = Q_i(\nu,\theta_i^*)/(\sigma_0 T_*^4); \quad \Phi_{i\nu}(\nu,\theta) = E_{\nu i}(\nu,\theta)/(\sigma_0 T_*^4);$$

$$E_{\nu i} = 2\pi h\nu^3 n^2 (\exp(h\nu/T_*\theta_i) - 1)^{-1}/c_0^2;$$

$$R(\theta) = \rho c L^2; \quad \theta(x,t) = T(x,t)/T_*; \quad \theta_i = T_i/T_* \quad (i = 1,2).$$

The Green function $\Gamma(x,z)$ by means of which the boundary-value problem (9)–(11) was reduced to the nonlinear integral equation (12) in the unknown temperature $\theta(x,t)$ is written as

$$\Gamma(x,z)=\begin{cases}-\cosh(x)\cosh(1-z)/\sinh(1), & x\leqslant z,\\ -\cosh(1-x)\cosh(z)/\sinh(1), & x\geqslant z.\end{cases}$$

The divergence of the radiative flux dE_ν/dx is expressed via the radiation intensities I_ν^+ and I_ν^-, which are determined from the solution of the boundary-value problem for the transfer equation obtained by the method of variation of an arbitrary constant and have the form

$$I_\nu^+(x,\mu)=\Big\{I_\nu^+(0,\mu)+\frac{n_\nu^2}{\mu}\int\limits_0^x k_\nu(y)I_{p\nu}(y)\exp\Big(\frac{1}{\mu}\int\limits_0^y k_\nu(z)\,dz\Big)\,dy\Big\}\exp\Big(-\frac{1}{\mu}\int\limits_0^x k_\nu(z)\,dz\Big); \qquad (13)$$

$$I_\nu^-(x,\mu)=\Big\{I_\nu^-(1,\mu)+\frac{n_\nu^2}{\mu}\int\limits_x^1 k_\nu(y)I_{p\nu}(y)\exp\Big(\frac{1}{\mu}\int\limits_y^1 k_\nu(z)\,dz\Big)\,dy\Big\}\exp\Big(-\frac{1}{\mu}\int\limits_x^1 k_\nu(z)\,dz\Big). \qquad (14)$$

Explicit expressions for the boundary intensities are derived from the solution of a system of two algebraic equation in $I_\nu^+(0,\mu)$ and $I_\nu^-(1,\mu)$ with the use of relations (7), (8), (13), and (14). Substituting the expressions for $I_\nu^+(0,\mu)$ and $I_\nu^-(1,\mu)$ into (13) and (14), we obtain the final expressions for the intensities. These expressions are used in determining the divergence of the radiative flux dE_ν/dx, whose integral spectral value is substituted into the energy equation (1).

Thus, the RCHT problem (1)–(8) in a flat layer of a selectively absorbing and radiating medium reduces to the solution of the nonlinear integral equation (12) in the desired dimensionless temperature $\theta(x,t)$. The method of solving Eq. (12) offers the possibility of using Newton–Kantorovich-type iteration processes and deriving a solution of the problem with any degree of accuracy.

A program of numerical solution of the unsteady energy equation in a heat-conducting, radiating, and absorbing medium was prepared on the basis of the developed algorithm. At each time step, the integral equation (12) was solved by the Newton–Kantorovich method [6].

The integrals in (12)–(14) were calculated by Gaüss quadrature formulas with 20 nodes. The derivative $\partial\theta/\partial t$ was approximated by a finite-difference ratio. For each moment of time, the temperature profile and the flux density of the total radiation were calculated. Results of the numerical solution of the integral equation (12) are given in Figs. 1–6. Calculations were performed for the following thermophysical and optical data in conformity with Plexiglas of thickness $L = 0.024$ m: $\lambda = 0.189$ W/(m·K), $T_0 = 300$ K, $T_* = 1600$ K, $T_1^* = 1000$ K, $T_2^* = 1000$ K, $n = 1.6$, and $a = 9$ m^2/sec (λ is the thermal conductivity, n is the refractive index, and a is the diffusivity). The spectral coefficient of volume absorption at $T = 300$ K was calculated on the basis of the experimentally measured transmission spectrum of SO-120 Plexiglas [7].

Figures 1–5 show the distribution of the dimensionless temperature across a glass layer at various moments of time. Here Bi_1, Bi_2, r_1, and r_2 are the dimensionless coefficients of convective heat transfer and the reflection coefficients, respectively.

The dynamics of layer heating is shown in Fig. 1, where the layer surface $x = 0$ is maintained at constant temperature ($\theta_1 = 0.38$ and $\mathrm{Bi}_1 = \infty$) and the layer surface $x = 1$ undergoes radiative-convective heating ($q_2 \neq 0$ and $\mathrm{Bi}_2 = 5.6$). Here the dashed curves correspond to $r_1 = 1$ and $r_2 = 1$, and the solid curves refer to $r_1 = 1$ and $r_2 = 0.5$. Figure 2 shows the temperature distribution in the layer where the surface $x = 0$ is heated owing to convection alone ($\mathrm{Bi}_1 = 5.6$). The conditions on the surface $x = 1$ are similar to those in Fig. 1. Figure 3 characterizes a heating process during which both surfaces of the layer are heated by radiative-convective fluxes ($q_1 \neq 0$ and $q_2 \neq 0$). Figure 4 shows how the temperature level in the layer increases markedly as the capacity for reflection of the layer surfaces decreases slightly. The dashed curves in Fig. 3 refer to $r_1 = r_2 = 1$, and the solid curves refer to $r_1 = 1$ and $r_2 = 0.5$. In Fig. 4, the dashed curves refer to $r_1 = 1$ and $r_2 = 0.8$, and the solid curves refer to $r_1 = 0.8$ and $r_2 = 0.5$.

The effect of the incident radiative flux on the temperature distribution in the glass layer for both surfaces is illustrated in Fig. 5 ($\mathrm{Bi}_1 = 0.56$ and $\mathrm{Bi}_2 = 0$ for $r_1 = 0.8$ and $r_2 = 1$; the solid curves refer to $q_1 = 13.6$ and $q_2 = 0$, and the dashed curves refer to $q_1 = 3.8$ and $q_2 = 0$).

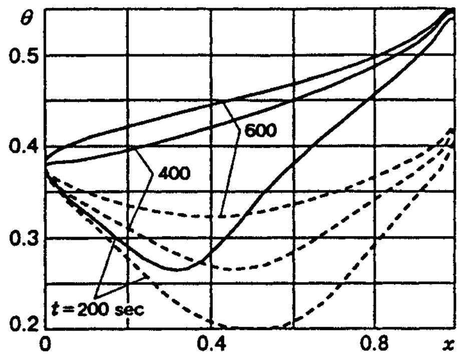

Fig. 1

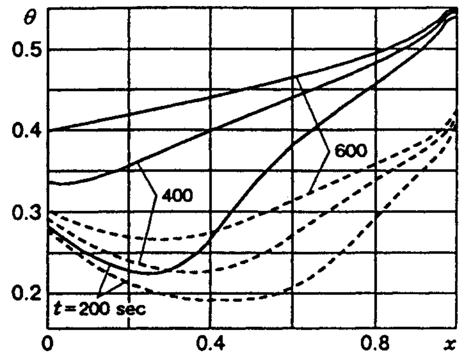

Fig. 2

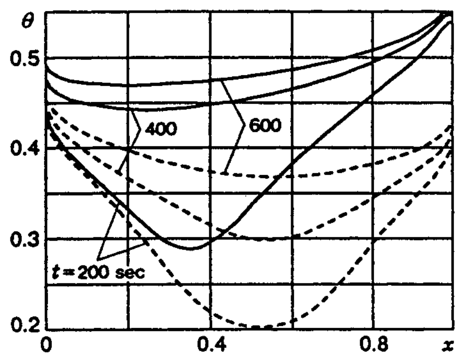

Fig. 3

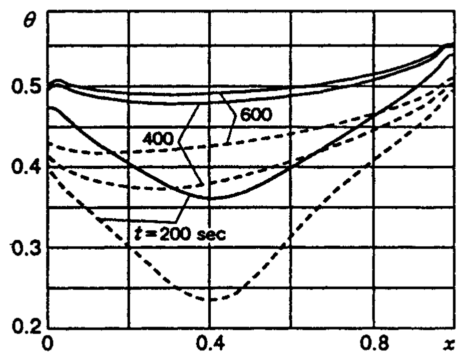

Fig. 4

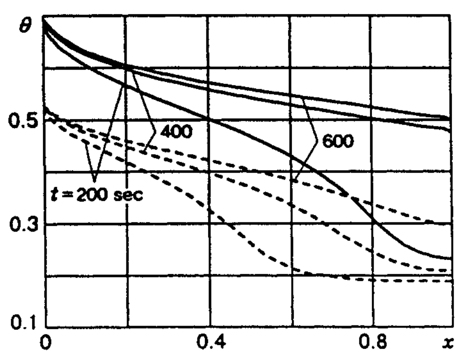

Fig. 5

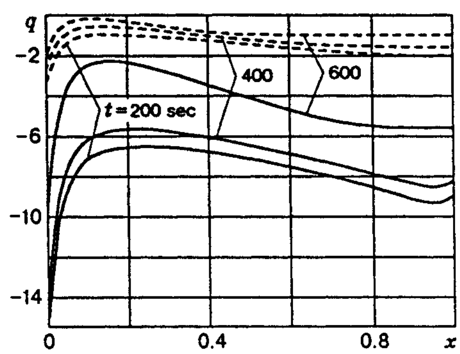

Fig. 6

The distribution of the radiative flux in the glass layer is shown in Fig. 6 for conditions similar to the previous case (Fig. 5).

In concluding, it is worth noting that the optical properties of the walls exert a significant effect on the formation of the temperature field in the Plexiglas layer. The intensity of heating of the layer depends on the incident radiative fluxes to a greater extent than on the convective fluxes.

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